The Quantum Viscosity Bound In Lovelock Gravity

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Abstract

Based on the finite-temperature AdS/CFT correspondence, we calculate the ratio of shear viscosity to entropy density in any Lovelock theories to any order. Our result shows that any Lovelock correction terms except the Gauss-Bonnet term have no contribution to the value of η/s. This result is consistent with that of Brustein and Medved's prediction.

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Stimulated by the conjecture of AdS/CFT correspondence [1–3], string theory has attracted a lot of attention, especially after the discovery that some theoretical results of the dual theory are consistent with that of the RHIC experiment, say, the ratio of the viscosity to the entropy density [4,5]. Recently, it was conjectured, based on the AdS/CFT correspondence, that for all possible nonrelativistic fluids, there may exist a universal lower bound (the KSS bound) on the viscosity/entropy-density ratio (we set $G = c = \hbar = k_B = 1$) [6]

$$\frac{\eta}{s} = \frac{1}{4\pi}. \tag{1}$$

This bound received great supports from several kinds of field theories [7–9], as well as the case with chemical potential in the theory [10, 11]. However, more recent work on the higher derivative gravity theories (see [12–18]) showed that the KSS bound is violated when the dual gravity is enlarged to include a stringy correction (see [19] for more about the KSS bound in higher derivative gravity). This correction is frequently referred to as the quantum correction, since in CFT side this is a correction of the 't Hooft coupling $\lambda = g_{YM}^2 N_c$. It is of particular significance to consider the $1/\lambda$ correction when we are dealing with non-extremely strong coupling fluids.

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Recently, the authors of [20] predicted that all Lovelock terms higher than the second order(the Gauss-Bonnet term) do NOT contribute to the value of η/s at all, and this prediction was partially confirmed in [21] for the third-order Lovelock gravity. In this paper we calculate the viscosity/entropy-density ratio directly in the Lovelock theory to any order, trying to make a complete verification of the prediction, and indeed, our result provides a direct support of this prediction as will see below.

We start with the Lovelock theory of gravity. This is one of the most general second order gravity theories in higher dimensional spacetimes and is free of ghost when expanding on a flat space [22] and hence is of particular interest. The Lagrangian density for general Lovelock gravity in D dimensions is $\mathcal{L} = \sum_{m=0}^{[D/2]} c_m \mathcal{L}_m$, where $\mathcal{L}_m$ is given by [23]

$$\mathcal{L}_m = \frac{1}{2^m}\sqrt{-g}\delta^{\lambda_1\sigma_1\cdots\lambda_m\sigma_m}_{\rho_1\kappa_1\cdots\rho_m\kappa_m} R_{\lambda_1\sigma_1}{}^{\rho_1\kappa_1}\cdots R_{\lambda_m\sigma_m}{}^{\rho_m\kappa_m}\,, \tag{2}$$

c_m is the m'th order coupling constant, $[D/2]$ denotes the integer value of $D/2$ and the Greek indices λ, ρ, σ and κ go from 0 to $D-1$. The symbol $R_{\lambda\sigma}{}^{\rho\kappa}$ is the Riemann tensor in D-dimensions and $\delta^{\lambda_1\sigma_1\cdots\lambda_m\sigma_m}_{\rho_1\kappa_1\cdots\rho_m\kappa_m}$ is the generalized totally antisymmetric Kronecker delta. The term $\mathcal{L}_0 = \sqrt{-g}$ is the cosmological term, while $\mathcal{L}_1 = \sqrt{-g}\delta^{\lambda_1\sigma_1}_{\rho_1\kappa_1} R_{\lambda_1\sigma_1}{}^{\rho_1\kappa_1}/2$ is the Einstein term. In general $\mathcal{L}_m$ is the Euler class of a $2m$ dimensional manifold.

Variation of the Lagrangian with respect to the metric yields the Lovelock equation of motion

$$0 = \mathcal{G}^{\nu}_{\mu} = -\sum_{m=0}^{[D/2]} \frac{c_m}{2^{(m+1)}}\delta^{\nu\lambda_1\sigma_1\cdots\lambda_m\sigma_m}_{\mu\rho_1\kappa_1\cdots\rho_m\kappa_m} R_{\lambda_1\sigma_1}{}^{\rho_1\kappa_1}\cdots R_{\lambda_m\sigma_m}{}^{\rho_m\kappa_m}\,, \tag{3}$$

As is shown in [24], there exist static exact solutions of Lovelock equation. Let us consider the following metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\sum_{i,j}^{D-2}\gamma_{ij}dx^i dx^j, \tag{4}$$

where $\gamma_{ij}dx^i dx^j$ represents the line element of a $(D-2)$-dimensional Einstein space. With this ansatz, we have

$$\mathcal{R}_{ijkl} = \kappa(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk}), \quad \mathcal{R}_{ij} = \kappa(D-3)\gamma_{ij}, \quad \mathcal{R} = \kappa(D-2)(D-3). \tag{5}$$

where κ is the curvature constant, whose value determines the geometry of the horizon. Without loss of the generality, one may take $\kappa = 1$, 0, or -1 representing sphere, flat and hyperbolic respectively.

Using this metric ansatz, we can calculate Riemann tensor components as

$$R_{tr}{}^{tr} = -\frac{f''}{2}, \;\; R_{ti}{}^{tj} = R_{ri}{}^{rj} = -\frac{f'}{2r}\delta_i{}^j, \;\; R_{ij}{}^{kl} = \left(\frac{\kappa - f}{r^2}\right)\left(\delta_i{}^k\delta_j{}^l - \delta_i{}^l\delta_j{}^k\right) \; . \tag{6}$$

Substituting (6) into (3) derives a simple equation

$$W[\psi] \equiv \sum_{m=0}^{n} \tilde{c}_m \psi^m = \frac{\mu}{r^{D-1}}, \tag{7}$$

where $\psi = r^{-2}(\kappa - f)$, $\mu > 0$ is a constant of integration which is related to the ADM mass by

$$M = \frac{\mu V_{D-2}}{16\pi G_D} \; , \tag{8}$$

where V_{D-2} is the volume of the $(D-2)$-dimensional hypersurface and G_D is the Newton constant. In (7), we also defined $\tilde{c}_m \equiv \frac{(D-3)!}{(D-2m-1)!}c_m$ and n is an integer with $0 < n \leq [D/2]$. In this paper we are considering AdS black brane in Lovelock gravity, so we have $c_0 = -2\Lambda$ with the cosmological constant $\Lambda = -(D-1)(D-2)/2l^2$ and $c_1 = 1$.

We would like to extract some information from the Lovelock black brane, such as their thermodynamic properties. One quantity which is of particular interest is the entropy S. Generally speaking, one can obtain the entropy of a black hole in higher derivative theories by using the thermodynamic relation $S = -\partial F/\partial T$ with F the free energy and T the Hawking temperature. By doing so one finds that the entropy of the Lovelock black brane is given by [25]

$$S = \frac{V_{D-2} r_+^{D-2}}{4G_D} \sum_{m=1}^{n} \frac{m(D-2)}{(D-2m)} \tilde{c}_m (\kappa r_+^{-2})^{m-1}, \tag{9}$$

where r_+ is the event horizon of the black brane which is the positive root of $f(r_+) = 0$. In the present paper, we mainly focus on the case where $\kappa = 0$. In this case we have a simple formula for the entropy density of the Lovelock black brane

$$s = \frac{r_+^{D-2}}{4G_D}, \tag{10}$$

and now, r_+ is a solution of $\psi(r_+) = 0$. The Hawking temperature of this case is given by

$$T = \frac{(D-1)\tilde{c}_0}{4\pi} r_+. \tag{11}$$

In what following, we would like to see the waves generated by a metric perturbation of the background. Generally speaking, there are scalar, vector and tensor modes depending on

the rotation symmetry. In this paper, we only study tensor perturbations which is closely related to the shear viscosity as will see below.

We now add a small tensor perturbations to the solution (4)

$$\delta g_{ij} = r^2 \phi(t,r) h_{ij}(x^i) \ , \quad (others) = 0 \tag{12}$$

where $\phi(t,r)$ represents the dynamical degrees of freedom. Here, h_{ij} are defined by

$$\nabla^k \nabla_k h_{ij} = k^2 h_{ij} \ , \qquad \nabla^i h_{ij} = 0 \ , \quad \gamma^{ij} h_{ij} = 0. \tag{13}$$

Here, ∇^i denotes a covariant derivative with respect to γ_{ij} and k^2 is the eigenvalue playing a role of momentum.

With these definition, one can obtain the first order perturbation equation of the Lovelock equation (3) [26]

$$0 = \delta \mathcal{G}^\nu_\mu = -\sum_{m=1}^{k} \frac{a_m}{2^{(m+1)}} \delta^{\nu\lambda_1\sigma_1\cdots\lambda_m\sigma_m}_{\mu\rho_1\kappa_1\cdots\rho_m\kappa_m} R_{\lambda_1\sigma_1}{}^{\rho_1\kappa_1} \cdots R_{\lambda_{m-1}\sigma_{m-1}}{}^{\rho_{m-1}\kappa_{m-1}} \delta R_{\lambda_m\sigma_m}{}^{\rho_m\kappa_m}, \tag{14}$$

where $\delta R_{ab}{}^{cd}$ represents the first order variation of the Riemann tensor and we have introduced a new quantity $a_m = mc_m (m > 0)$. As shown in [26], it is straightforward once we know the expressions of quantities $\delta R_{ti}{}^{tj}$, $\delta R_{ri}{}^{rj}$ and $\delta R_{ij}{}^{kl}$. Then the calculation becomes a mathematical game and the result is ready-made [26]

$$0 = \delta \mathcal{G}_i{}^j = \frac{1}{r^{D-4}} \left[\frac{h}{2f} \left(\ddot{\phi} - f^2 \phi'' \right) - \left(\frac{(r^2 f h)'}{2r^2} \right) \phi' + \frac{(k^2 + 2\kappa) h'}{2(D-4) r} \phi \right] h_i{}^j, \tag{15}$$

where

$$\begin{aligned} h(r) &= \frac{d}{dr} \left[\frac{r^{D-3}}{D-3} \frac{dW[\psi]}{d\psi} \right] \\ &= r^{D-4} - \sum_{m=2}^{n} \left[\frac{m\tilde{c}_m r^{D-2m-2} (\kappa - f)^{m-2}}{D-3} \left\{ (m-1) r f' - (D-2m-1)(\kappa - f) \right\} \right] \end{aligned} \tag{16}$$

Using the Fourier decomposition

$$\phi(t,r) = \int \frac{d\omega}{2\pi} e^{-i\omega t} \phi(r), \tag{17}$$

we obtain the linearized equation of motion for $\phi(r)$:

$$\phi''(r) + \left(\frac{(r^2fh)'}{r^2fh}\right)\phi'(r) + \frac{1}{f^2}\left(\omega^2 - \frac{(k^2+2\kappa)fh'}{(D-4)rh}\right)\phi(r) = 0\ . \tag{18}$$

It is convenient to introduce a new dimensionless coordinate $u = (r_+/r)^{(D-1)/2}$ with r_+ the event horizon of the black brane. In this coordinate frame, $u = 0$ corresponds to the boundary and $u = 1$ the horizon. The linearized equation of motion (18) then becomes (for $\kappa = 0$)

$$\phi''(u) + \frac{g'(u)}{g(u)}\phi'(u) + \frac{\bar{\omega}^2}{u^{\frac{2D-6}{D-1}}\psi^2(u)}\phi(u) - \frac{D-1}{2(D-4)}\cdot\frac{h'\bar{k}^2}{u^{\frac{D-5}{D-1}}\psi(u)h}\phi(u) = 0 \tag{19}$$

where

$$g(u) = -r_+^{4-D}\psi(u)h(u)u^{\frac{D-7}{D-1}}, \tag{20}$$

$$\bar{\omega} \equiv \frac{2}{(D-1)r_+}\omega, \qquad \bar{k} \equiv \frac{2}{(D-1)r_+}k, \tag{21}$$

and the prime denotes the derivative with respect to u.

Now we shall calculate the shear viscosity in Lovelock gravity theories. Generally speaking, the shear viscosity η can be calculated via Kubo formula,

$$\eta = -\lim_{\omega\to 0}\frac{\mathrm{Im}(G^R(\omega,0))}{\omega}, \tag{22}$$

where G^R is the retarded Green's function

$$G^R(\omega,\vec{k}) = -i\int dt d\vec{x} e^{-i\vec{k}.\vec{x}}\theta(t) < [\hat{\mathcal{O}}(x)\hat{\mathcal{O}}(0)] >, \tag{23}$$

with $\hat{\mathcal{O}}$ some boundary CFT operators. According to AdS/CFT correspondence, the Green's function can be calculated from the dual gravity side via the Gubser-Klebanov-Polyakov/Witten relation [2, 3]

$$\langle e^{\int_{\partial M}\phi_0\hat{\mathcal{O}}}\rangle = e^{-S_{cl}[\phi_0]},$$

where ϕ is the bulk field and ϕ_0 is its value at the boundary, i.e., $\phi_0 = \lim_{u\to 0}\phi(u)$. Extracting the part of S_{cl} that is quadratic in ϕ and inserting the solution of the linearized field equation we may get a surface term in four dimensions by using the equation of motion,

$$S_{cl}[\phi_0] = \int\frac{d^{D-1}k}{(2\pi)^{D-1}}\phi_0(-k)G(k,u)\phi_0(k)\Big|_{u=0}^{u=1}, \tag{24}$$

where $u = (r_+/r)^{(D-1)/2}$ as defined previously. In this way, we obtain the following relation for the retarded Green's function [27]

$$G^R(k) = 2G(k,u)\Big|_{u=0}, \tag{25}$$

where the incoming boundary condition at the horizon is imposed. The shear viscosity then can be calculated by using (22).

In the following we would like to calculate the shear viscosity, following the procedures introduced above. The main task is to solve the equation of motion (19) in hydrodynamic regime *i.e.*, small ω and k. To solve the wave equation (19) we first examine the behavior around the horizon where $u = 1$. For this purpose it is convenient to impose a solution as

$$\phi(u) = (1-u)^{\nu} F(u), \tag{26}$$

with $F(u)$ regular at the horizon. Substituting (26) into the wave equation (19) and leaving the most divergent terms, we can obtain

$$\nu = \pm i \frac{\bar{\omega}}{\psi'(1)} \tag{27}$$

where we have used the relations

$$g(u \to 1) = -g'(1)(1-u) + \mathcal{O}((1-u)^2), \tag{28}$$
$$\psi(u \to 1) = -\psi'(1)(1-u) + \mathcal{O}((1-u)^2). \tag{29}$$

In present paper we choose "$-$" sign in eq. (27) for convenience.

To get the viscosity via Kubo formula (22), the standard procedure is to consider series expansion of the solution in terms of frequencies up to the linear order of ω,

$$F(u) = F_0(u) + \nu F_1(u) + \mathcal{O}(\nu^2, k^2). \tag{30}$$

Then the equation of motion (19) becomes the following form up to $\mathcal{O}(\nu)$,

$$[g(u)F'(u)]' - \nu \left(\frac{1}{1-u} g(u)\right)' F(u) - \frac{2\nu}{1-u} g(u) F'(u) = 0. \tag{31}$$

After substituting the series expansion (30) into the equation (31), we obtain the following equations of motion for $F_0(u)$ and $F_1(u)$

$$[g(u)F_0'(u)]' = 0, \tag{32}$$
$$[g(u)F_1'(u)]' - \left(\frac{1}{1-u} g(u)\right)' F_0(u) = 0. \tag{33}$$

By requiring that the functions $F_0(u)$ and $F_1(u)$ are regular at the horizon one gets the following results

$$F_0(u) = C, \tag{34}$$
$$F_1'(u) = \left(\frac{1}{1-u} + \frac{g'(1)}{g(u)}\right) C, \tag{35}$$

where again we have used the relation (28) and the constant C can be determined in terms of boundary value of the field, i.e., $C = \phi_0\Big(1 + \mathcal{O}(\nu)\Big)$.

Now we shall calculate the retarded Green's function. Using the equation of motion, the action reduces to the surface terms. The relevant part is given by

$$S_{cl}[\phi(u)] = -\frac{(D-1)r_+^{D-1}}{64\pi G_D}\int \frac{d^{D-1}k}{(2\pi)^{D-1}}\Big(g(u)\phi(u)\phi'(u) + \cdots\Big)\Bigg|_{u=0}^{u=1}. \tag{36}$$

Near the boundary $u = \varepsilon$, using the perturbative solution of $\phi(u)$, we get

$$\begin{aligned}\phi'(\varepsilon) &= \nu\frac{g'(1)}{g(\varepsilon)}\phi_0 + \mathcal{O}(\nu^2, k^2)\\ &= -i\frac{\bar{\omega}}{\psi'(1)}\frac{g'(1)}{g(\varepsilon)}\phi_0 + \mathcal{O}(\omega^2, k^2).\end{aligned} \tag{37}$$

Therefore we can read off the correlation function from the relation (25),

$$G^R(\omega, k) = i\omega\frac{1}{16\pi G_D}\left(\frac{r_+^{D-2}}{\psi'(1)}\right)g'(1) + \mathcal{O}(\omega^2, k^2), \tag{38}$$

where contact terms are subtracted. Then the shear viscosity can be obtained by using Kubo formula (22),

$$\eta = -\frac{1}{16\pi G_D}\left(\frac{r_+^{D-2}}{\psi'(1)}\right)g'(1). \tag{39}$$

The ratio of the shear viscosity to the entropy density is concluded as

$$\frac{\eta}{s} = -\frac{1}{4\pi}\frac{g'(1)}{\psi'(1)}. \tag{40}$$

From (20) we have a relation $g'(1) = -r_+^{4-D}\psi'(1)h(1)$ and $h(1)$ can be obtained from (16) by inserting $\kappa = 0$

$$h(1) = r_+^{D-4}\Big(1 - (D-1)(D-4)\tilde{c}_0 a_2\Big).$$

It is straightforward to show that

$$\frac{\eta}{s} = \frac{1}{4\pi}\Big(1 - (D-1)(D-4)\tilde{c}_0 a_2\Big) = \frac{1}{4\pi}\Big(1 - \frac{2(D-1)(D-4)\lambda}{l^2}\Big), \tag{41}$$

where we have defined $\lambda = c_2$. This result is exactly the one predicted in [20].

In summary, we have computed the ratio of shear viscosity to entropy density for any Lovelock theories. Our result shows that any correction terms except the Gauss-Bonnet term do not affect the value of η/s, and this confirms the prediction made by [20]. During our calculation, we have chosen a vanishing curvature constant κ. Actually, our result is

still valid (for leading term) for nonzero κ if we focus on a large black brane. In the large black brane limit, both the entropy and the viscosity have the same leading terms as those of $\kappa = 0$. This can be seen by noting the expressions of entropy density and viscosity. From (9), the entropy density of the Lovelock black brane with nonzero κ can be expanded, in the large black brane limit($i.e.$, $\frac{\kappa}{r_+^2} \ll 1$), to the first order as

$$s = \frac{r_+^{D-2}}{4G_D}\left[1 + \frac{2(D-2)\tilde{c}_2}{D-4}\cdot\frac{\kappa}{r_+^2}\right] + \mathcal{O}\left(\frac{\kappa}{r_+^2}\right). \tag{42}$$

With the same spirit one can also expand the shear viscosity to the first order in the large black brane limit. This can be done by repeating the previous procedures and noting that $h(1) = h(u=1)$ can be obtained from (16). In this way, the shear viscosity for nonvanishing κ can be expanded to the first order as

$$\begin{aligned}\eta = & \frac{r_+^{D-2}}{16\pi G_D}\left\{1 - \frac{2(D-1)(D-4)\lambda}{l^2} - \right.\\ & \left.\frac{2(D-1)}{D-3}\left[\tilde{c}_2(1-2\tilde{c}_2) + 3\frac{\tilde{c}_3}{l^2} - (D-5)\tilde{c}_2\right]\cdot\frac{\kappa}{r_+^2}\right\} + \mathcal{O}\left(\frac{\kappa}{r_+^2}\right). \end{aligned} \tag{43}$$

From (42) and (43) it is obvious that the leading terms of the entropy density and the viscosity for $\kappa \neq 0$ are the same as those of $\kappa = 0$. In other words, in the large black brane limit, the curvature constant κ has no contribution to the shear viscosity to entropy density ratio for the leading term. The sub-leading terms, however, receive contributions from κ.

So far we are confident with the violation of the KSS bound while we are not sure the existence of a universal lower bound of η/s. A great progress alone this line appeared several months ago when the authors of [28] gave a proof for the existence of a universal bound of η/s for any ghost-free extension of Einstein theory. However, the work in [14] shows that the causality violation of the dual gauge theory may put constraints on the coefficients of higher derivative terms and this in turn will put constraints on the value of η/s. Then it is natural to ask if the lower bound still exists as these constraints are taken into account. Recent progress made by Camanho and Edelstein in [29] provides us with an answer that the causality violation, as expected, may impose a constraint on the bound of the η/s at least for cubic Lovelock gravity. For completeness, we briefly catch some important results from [29], so as to compare the lower limit on η/s from causality violation and the result in the present paper. Actually, the formula of the ratio between η and s obtained in [29] is not different from our result (41). What is new of their result is that by imposing a condition so that the causality violation can be avoided, they found constraints on the coefficient λ (or c_2 as defined) in (41). For any order ($n \geq 2$) Lovelock gravity, the condition to be free of causality is that

$$\sum_{m=1}^{n} m c_m \Lambda^{m-1}\left(1 + \frac{\gamma(m-1)(D-1)}{D-3}\right) \geq 0, \tag{44}$$

where $\gamma = -2, -1, 2/(D-4)$ represent helicity zero, helicity one and helicity two graviton, respectively. Therefore, though any correction terms higher than the second order of Lovelock gravity do not manifestly contribute to the ratio of viscosity to entropy density, it does not mean that they are irrelevant to this ratio. Through (44) we see these terms impose a constraint on the value of c_2 (or λ) thus in turn affecting the lower bound for η/s.

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